Chapter 2 Curriculum Framework

This curriculum is adapted from the *Syllabuses for Secondary Schools – Pure Mathematics (Advanced Level) 1992* (referred as *Syllabus 1992* hereafter). Some topics have been deleted or trimmed from the *Syllabus 1992*. The relevant changes and the comparison between this curriculum and the *Syllabus 1992* can be found in Appendices 1 and 2 respectively. The rationale of the revision is to create curriculum space for consolidating concepts and adjusting teaching strategies (to cater for students' individual differences), etc. so as to improve the learning of AL Pure Mathematics. The total teaching time for this curriculum should be unchanged when compared with the *Syllabus 1992* to serve the said rationale (refer to the suggested time allocation on page 8).

Instead of dividing the contents of the curriculum into dimensions as in the secondary mathematics curriculum, they are divided into 2 topic areas, namely "Algebra" and "Calculus and Analytical Geometry". "Algebra" consists of 9 units while "Calculus and Analytical Geometry" 7 units. For each unit, specific learning objectives are given to provide a sharper focus. The subject matter of each unit is broken into sub-units to facilitate learning/teaching and achievement of the specific learning objectives.

Some of the contents in the curriculum may overlap with those in the Additional Mathematics Curriculum, but it should be noted that they may have different approaches and depths of treatment. Teachers should also note that knowledge of the contents of the Additional Mathematics Curriculum is not required in studying AL Pure Mathematics.

Content and Specific Learning Objectives

Unit	Content	Specific Learning Objectives
A1	**The Language of Mathematics** 1.1 Set Language 1.2 Simple Logic	1. To understand the first notion of set language 2. To understand the first notion of logic
A2	**Functions** 2.1 Functions and their graphs 2.2 Properties and operations of	1. To recognize function as a fundamental tool in other branches of mathematics

	functions 2.3 Algebraic functions 2.4 Trigonometric functions and their formulae 2.5 Exponential and logarithmic functions	2. To sketch and to describe the shapes of different functions
A3	**Mathematical Induction** 3.1 The Principle of Mathematical Induction and its applications 3.2 Other common variations of the Principle of Mathematical Induction and their applications	1. To understand the Principle of Mathematical Induction 2. To apply the Principle of Mathematical Induction to prove propositions involving integers 3. To be able to modify the Principle of Mathematical Induction to suit different purposes
A4	**Inequalities** 4.1 Absolute inequalities 4.2 A.M. ≥ G.M. 4.3 Cauchy-Schwarz's inequality 4.4 Conditional inequalities	1. To learn the elementary properties of inequalities 2. To prove simple absolute inequalities 3. To solve simple conditional inequalities
A5	**The Binomial Theorem for Positive Integral Indices** 5.1 The binomial theorem for positive integral indices 5.2 Applications of the binomial theorem for positive integral indices 5.3 Simple properties of the binomial coefficients	1. To learn and apply the binomial theorem for positive integral indices 2. To study the simple properties of the binomial coefficients
A6	**Polynomials and Equations** 6.1 Polynomials with real coefficients in one variable 6.2 Rational functions 6.3 Polynomial equations with real coefficients in one variable	1. To learn the properties of polynomials with real coefficients in one variable 2. To learn division algorithm, remainder theorem and Euclidean algorithm and their applications 3. To resolve rational functions into partial fractions 4. To learn the properties of roots of polynomial equations with real coefficients in one variable

A7	**Vectors in** R^2 **and** R^3 *(deleted)*	
A8	**Matrices** 8.1 Matrices and their operations 8.2 Square matrices of order 2 and 3 8.3 Applications to two dimensional geometry	1. To learn the concept and operations of matrices 2. To learn the properties and operations of square matrices of order 2 and 3 and determinants 3. To apply matrices to two dimensional geometry
A9	**System of Linear Equations in 2 or 3 Unknowns** 9.1 Gaussian elimination and Echelon form 9.2 Existence and uniqueness of solution	1. To solve a system of linear equations using Gaussian elimination 2. To recognize the existence and uniqueness of solution
A10	**Complex Numbers** 10.1 Definition of complex numbers and their arithmetic operations 10.2 Argand diagram, argument and conjugate 10.3 Simple applications in plane geometry 10.4 De Moivre's theorem	1. To learn the properties of complex numbers, their geometrical representations and applications 2. To learn the De Moivre's Theorem and its applications in finding the nth roots of complex numbers, in solving polynomial equations and proving trigonometric identities
B1	**Sequence, Series and their Limits** 1.1 Sequence and series 1.2 Limit of a sequence and series 1.3 Convergence of a sequence and series	1. To learn the concept of sequence and series 2. To understand the intuitive concept of the limit of sequence and series 3. To understand the behaviour of infinite sequence and series
B2	**Limit, Continuity and Differentiability** 2.1 Limit of a function 2.2 Continuity of a function 2.3 Differentiability of a function	1. To understand the intuitive concept of the limit of a function 2. To understand the intuitive concept of continuity and differentiability of a function 3. To recognize limit as a fundamental concept in calculus
B3	**Differentiation** 3.1 Fundamental rules for differentiation	1. To acquire different techniques of differentiation

	3.2 Differentiation of trigonometric functions 3.3 Differentiation of composite functions and inverse functions 3.4 Differentiation of implicit functions 3.5 Differentiation of parametric equations 3.6 Differentiation of logarithmic and exponential function 3.7 Higher order derivatives and Leibniz's Theorem 3.8 The Rolle's Theorem and Mean Value Theorem	2. To learn and acquire techniques to find higher order derivative 3. To understand the intuitive concept of Rolle's Theorem and Mean Value Theorem
B4	**Application of Differentiation** 4.1 The L' Hospital's Rule 4.2 Rate of change 4.3 Monotonic functions 4.4 Maxima and minima 4.5 Curve sketching	1. To learn and to use the L' Hospital's Rule 2. To learn the applications of differentiation
B5	**Integration** 5.1 The Riemann definition of integration 5.2 Simple properties of definite integrals 5.3 The Mean Value Theorem for Integrals 5.4 Fundamental Theorem of Integral Calculus and its application to the evaluation of integrals 5.5 Indefinite integration 5.6 Method of integration 5.7 Improper integrals *(deleted)*	1. To understand the notion of integral as limit of a sum 2. To learn some properties of integrals 3. To understand the Fundamental Theorem of Integral Calculus 4. To apply the Fundamental Theorem of Integral Calculus in the evaluation of integrals 5. To learn the methods of integration
B6	**Application of Integration** 6.1 Plane area 6.2 Arc length *(deleted)* 6.3 Volume of revolution 6.4 Area of surface of revolution *(deleted)* 6.5 Limit of sum	1. To learn the application of definite integration in the evaluation of plane area and volume of solid of revolution 2. To apply definite integration to the evaluation of limit of sum

B7	**Analytical Geometry** 7.1 Basic knowledge in coordinate geometry 7.2 Sketching of curves in the polar coordinate system *(deleted)* 7.3 Conic sections in rectangular coordinate system 7.4 Tangents and normals of conic sections 7.5 Locus problems in rectangular coordinate system 7.6 Tangents and normals of plane curves	1. To learn the conic sections 2. To study locus problems algebraically 3. To solve related problems

Suggested Sequence

There are two main topic areas in the curriculum and they are presented in the sequence as below.

Topic Area A **Algebra**		**Topic Area B** **Calculus and Analytical Geometry**	
Unit	**Content**	**Unit**	**Content**
A1	The Language of Mathematics	B1	Sequence, Series and their Limits
A2	Functions	B2	Limit, Continuity and Differentiability
A3	Mathematical Induction	B3	Differentiation
A4	Inequalities	B4	Application of Differentiation
A5	The Binomial Theorem for Positive Integral Indices	B5	Integration
A6	Polynomials and Equations	B6	Application of Integration
A8	Matrices	B7	Analytical Geometry
A9	System of Linear Equations in 2 or 3 Unknowns		
A10	Complex Numbers		

(Note: The unit A7 has been deleted from the *Syllabus 1992*.)

Teachers should note that the sequence presented here only serves as an example and the categorization of the topics as A or B is done with a belief that such grouping and arrangement may offer a certain degree of fluency in teaching. In fact, teachers are free to design their own teaching sequence to suit the needs of their students. When designing a school-based curriculum of the subject, teachers should ensure that the curriculum should be coherent and students have already possessed the pre-requisite knowledge for the topics concerned. One possible sequence is as follows:

$$\begin{array}{ccccccccccccc} A1 & \longrightarrow & A2 & \longrightarrow & A3 & \longrightarrow & A4 & \longrightarrow & A5 & \longrightarrow & A6 & \longrightarrow & B1 \\ \longrightarrow & B2 & \longrightarrow & B3 & \longrightarrow & B4 & \longrightarrow & B5 & \longrightarrow & B6 & \longrightarrow & B7 & \longrightarrow \\ & & & & A8 & \longrightarrow & A9 & \longrightarrow & A10 & & & & \end{array}$$

Some teachers, on the other hand, may prefer to apportion the number of periods allotted per week/cycle and start teaching according to the two sequences of topics in a “parallel” manner. Amongst different feasible approaches and sequencing of topics, teachers are expected to exercise their expertise in smoothing out, during teaching, possible irregularities sprung from the teaching sequence preferred. It is anticipated and advisable that the unit A1 “The Language of Mathematics” should be taught in the first place as a preliminary prerequisite so as to familiarize students with the usual symbols and trends of thinking in AL Pure Mathematics. The presentation in this curriculum and assessment guide will provide teachers with maximum flexibility so that the course of teaching adopted can be adjusted to meet the individual teaching situation.

To realize the spirit of the curriculum, teachers are advised to teach the curriculum as a connected body of mathematical knowledge as far as possible. Adequate arrangements should be provided for students to inquire, reason and communicate mathematically.

Suggested Time Allocation

The suggested time allocation for the course is 8 periods per week. It is assumed that there are 40 minutes in each period and 5 days in a week. A total of 380 periods (excluding the time spent on classroom tests and examinations) should be available for the two years. A time ratio is given to aid teachers in judging how far to take a given topic. This time ratio will indicate what fraction of the available total time may be spent on a certain unit, but schools are free to choose an equivalent or slightly different time allocation to suit their own situations. It can be seen, from the following table, that the total time ratio 312 is still 68

periods running short. This amount of time could be used for carrying out exploratory activities, consolidation activities or enrichment activities, etc. to suit the teaching approaches and the standard of students in the individual schools.

Topic Area A Algebra			**Topic Area B Calculus and Analytical Geometry**		
Unit	**Content**	**Time Ratio**	**Unit**	**Content**	**Time Ratio**
A1	The Language of Mathematics	10	B1	Sequence, Series and their Limits	18
A2	Functions	28	B2	Limit, Continuity and Differentiability	13
A3	Mathematical Induction	11	B3	Differentiation	28
A4	Inequalities	20	B4	Application of Differentiation	20
A5	The Binomial Theorem for Positive Integral Indices	13	B5	Integration	41
A6	Polynomials and Equations	15	B6	Application of Integration	13
A8	Matrices	21	B7	Analytical Geometry	27
A9	System of Linear Equations in 2 or 3 Unknowns	10			
A10	Complex Numbers	24			
	Sub-Total	152		Sub-Total	160
				Total	**312**

The following table shows the detailed breakdown of the units and the corresponding time ratios:

Unit	**Content**	Time Ratio	Unit Total
A1	**The Language of Mathematics**		
	1.1 Set Language	5	
	1.2 Simple Logic	5	10
A2	**Functions**		
	2.1 Functions and their graphs	2	
	2.2 Properties and operations of functions	4	
	2.3 Algebraic functions	2	
	2.4 Trigonometric functions and their formulae	14	
	2.5 Exponential and logarithmic functions	6	28

A3	**Mathematical Induction**		
	3.1 The Principle of Mathematical Induction and its applications	6	
	3.2 Other common variations of the Principle of Mathematical Induction and their applications	5	11
A4	**Inequalities**		
	4.1 Absolute inequalities	6	
	4.2 A.M. ≥ G.M.	4	
	4.3 Cauchy-Schwarz's inequality	3	
	4.4 Conditional inequalities	7	20
A5	**The Binomial Theorem for Positive Integral Indices**		
	5.1 The binomial theorem for positive integral indices	3	
	5.2 Applications of the binomial theorem for positive integral indices	5	
	5.3 Simple properties of the binomial coefficients	5	13
A6	**Polynomials and Equations**		
	6.1 Polynomials with real coefficients in one variable	5	
	6.2 Rational functions	4	
	6.3 Polynomial equations with real coefficients in one variable	6	15
A7	**Vectors in** R^2 **and** R^3 *(deleted)*	/	/
A8	**Matrices**		
	8.1 Matrices and their operations	4	
	8.2 Square matrices of order 2 and 3	9	
	8.3 Applications to two dimensional geometry	8	21
A9	**System of Linear Equations in 2 or 3 Unknowns**		
	9.1 Gaussian elimination and Echelon form	5	
	9.2 Existence and uniqueness of solution	5	10
A10	**Complex Numbers**		
	10.1 Definition of complex numbers and their arithmetic operations	3	
	10.2 Argand diagram, argument and conjugate	6	
	10.3 Simple applications in plane geometry	5	
	10.4 De Moivre's theorem	10	24
		Sub-Total	152

Unit	Content	Time Ratio	Unit Total
B1	**Sequence, Series and their Limits**		
	1.1 Sequence and series	6	
	1.2 Limit of a sequence and series	7	
	1.3 Convergence of a sequence and series	5	18
B2	**Limit, Continuity and Differentiability**		
	2.1 Limit of a function	5	
	2.2 Continuity of a function	4	
	2.3 Differentiability of a function	4	13
B3	**Differentiation**		
	3.1 Fundamental rules for differentiation	4	
	3.2 Differentiation of trigonometric functions	2	
	3.3 Differentiation of composite functions and inverse functions	4	
	3.4 Differentiation of implicit functions	2	
	3.5 Differentiation of parametric equations	2	
	3.6 Differentiation of logarithmic and exponential function	6	
	3.7 Higher order derivatives and Leibniz's Theorem	5	
	3.8 The Rolle's Theorem and Mean Value Theorem	3	28
B4	**Application of Differentiation**		
	4.1 The L' Hospital's Rule	4	
	4.2 Rate of change	3	
	4.3 Monotonic functions	2	
	4.4 Maxima and minima	5	
	4.5 Curve sketching	6	20
B5	**Integration**		
	5.1 The Riemann definition of integration	5	
	5.2 Simple properties of definite integrals	4	
	5.3 The Mean Value Theorem for Integrals	2	
	5.4 Fundamental Theorem of Integral Calculus and its application to the evaluation of integrals	4	
	5.5 Indefinite integration	6	
	5.6 Method of integration	20	
	5.7 Improper integrals *(deleted)*	/	41

B6	**Application of Integration**		
	6.1 Plane area	5	
	6.2 Arc length *(deleted)*	/	
	6.3 Volume of revolution	4	
	6.4 Area of surface of revolution *(deleted)*	/	
	6.5 Limit of sum	4	13
B7	**Analytical Geometry**		
	7.1 Basic knowledge in coordinate geometry	5	
	7.2 Sketching of curves in the polar coordinate system *(deleted)*	/	
	7.3 Conic sections in rectangular coordinate system	7	
	7.4 Tangents and normals of conic sections	6	
	7.5 Locus problems in rectangular coordinate system	5	
	7.6 Tangents and normals of plane curves	4	27
		Sub-Total	160

Total (Topic Areas A and B)	**312**